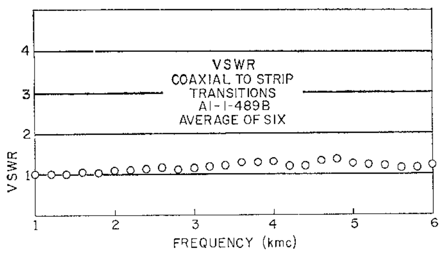

Fig. 2.

Our experience has shown that these connectors have a low enough VSWR to be adequate for all but the most critical situations and may be assembled and dismantled numerous times without damage to either the connector itself or to the strip to which it has been connected.

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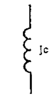
A Note on Loaded Line Synthesis*

The purpose of this note is to present an alternate derivation of a formula for the synthesis of a loaded line. The problem is to determine the values of the normalized susceptances a and c mounted an exact quarter-wavelength apart along a uniform line. These are arranged in the order $a, c, c, \cdots c, a$ to achieve a loaded line with a given phase shift and perfect match. In order to analyze a particular loaded line design for standing-wave ratio and phase shift over a band of frequencies on a digital computer, it is worthwhile to know the values of the susceptances to many more decimal places than one would achieve from a simple graph.

In branch transmission line couplers, where the length of branches and the spacing between them are each exactly a quarter-wavelength long, the calculated response (standing-wave ratio and fraction of power out each arm) is symmetrical on a normalized frequency basis F where $F=f/f_0$ or $F=\lambda g_0/\lambda g$ when waveguide is used. For example, the response is the same at $F=0.80$ as it would be at $F=1.20$.

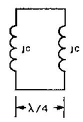
The response is not symmetrical in the case of a half-wave or quarter-wave rotating plate consisting of round waveguide loaded with a cascade of irises which are alternately all capacitive or all inductive as the guide is rotated. However, if the values of a and c are assumed to have a frequency variation which is reasonable (inductive varying as $1/F$ and capacitive varying as F), the difference in phase between the two positions of the plate does not change with frequency near the design center. That is, the phase shift of the plate, when it is used as a capacitive iris array minus that when it is an inductive iris array, has the desired value, and the rate of change of this value with respect to frequency is zero at the design center. The assumption is made that the magnitude of the susceptances are the same when the irises are inductive as when the irises are capacitive.

Consider a single shunt susceptance of normalized value, jc. Its matrix is



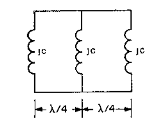
$$M = \begin{bmatrix} 1 & 0 \\ jc & 1 \end{bmatrix}. \quad (1)$$

For two such susceptances separated by a quarter wavelength of unity impedance line



$$M = \begin{bmatrix} 1 & 0 \\ jc & 1 \end{bmatrix} \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jc & 1 \end{bmatrix} = \begin{bmatrix} -c & j1 \\ -j(c^2-1) & -c \end{bmatrix}. \quad (2)$$

For three susceptances



$$M = \begin{bmatrix} c^2-1 & j(-c) \\ -j(-c^3+2c) & c^2-1 \end{bmatrix}. \quad (3)$$

For n susceptances separated by $n-1$ quarter wavelength of line

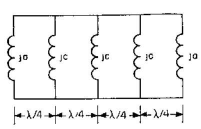
$$M = \begin{bmatrix} S_{n-1}(-c) & jS_{n-2}(-c) \\ -jS_n(-c) & S_{n-1}(-c) \end{bmatrix} \quad (4)$$

where

$$\begin{aligned} S_{-1}(-c) &= 0 \\ S_0(-c) &= 1 \\ S_1(-c) &= -c \qquad (5) \\ S_2(-c) &= c^2-1 \\ S_3(-c) &= -c^3+2c \\ S_{n+1}(-c) &= -cS_n(-c) - S_{n-1}(-c). \qquad (6) \end{aligned}$$

The polynomials $S_n(-c)$ are Chebyshev [2] polynomials of the second kind.

If a shunt susceptance ja is put a quarter-wavelength away from each end to achieve a matched condition, the matrix becomes



$$M = \begin{bmatrix} -aS_n(-c) - S_{n-1}(-c) & jS_n(-c) \\ -j(a^2S_n(-c) + 2aS_{n-1}(-c) + S_{n-2}(-c)) & -aS_n(-c) - S_{n-1}(-c) \end{bmatrix}. \quad (7)$$

This matrix represents a line of length θ and a normalized impedance of unity if each term in it is equal to those of a matrix of this length of line:

$$M = \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix}. \quad (8)$$

On equating matrix (7) with (8), the value of c can be found by setting $S_n(-c) = \sin\theta$ by inverse interpolation in the tables [2]. Then the value of a to give a matched line is

$$a = \left| \frac{|\cos\theta| - |S_{n-1}(-c)|}{\sin\theta} \right|. \quad (9)$$

There are two values of a which will match out the loaded line; the one which should usually be used is that with the smaller magnitude. The algebraic sign of a is the same as c and may be either positive or negative, depending on whether the resulting loaded line is to be effectively longer or shorter than the unloaded line.

An earlier contribution [3] has been based on using the value of a as exactly one-half that of c. The element spacing was then computed so that the differential phase shift was as desired. This scheme results in spacings which are not an exact quarter-wavelength and therefore may not give a symmetrical frequency response especially for small numbers of elements.

Appendix

The general circuit matrix of a dissipationless reciprocal symmetrical two-port network can be represented as

$$\begin{bmatrix} A & B \\ C & A \end{bmatrix}. \quad (10)$$

In this $A^2-BC=1$ and A is a purely real quantity and B and C are purely imaginary. The matrix of a cascade of n of these networks may be found by raising (1) to the nth power.

For $n=2$

$$\begin{bmatrix} A & B \\ C & A \end{bmatrix}^2 = \begin{bmatrix} 2A^2-1 & B(2A) \\ C(2A) & 2A^2-1 \end{bmatrix}. \quad (11)$$

For $n=3$

$$\begin{bmatrix} A & B \\ C & A \end{bmatrix}^3 = \begin{bmatrix} 4A^3-3A & B(4A^2-1) \\ C(4A^2-1) & 4A^3-3A \end{bmatrix}. \quad (12)$$

For the general case of n identical networks the matrix product may be expressed as [5]

$$M^n = \begin{bmatrix} T_n(A) & BU_{n-1}(A) \\ CU_{n-1}(A) & T_n(A) \end{bmatrix}. \quad (13)$$

$T_n(A)$ and $U_{n-1}(A)$ are Chebyshev polynomials of the first and second kind in A [2].

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An alternate expression for this matrix product is

$$M^n = \begin{bmatrix} 1/2C_n(2A) & BS_{n-1}(2A) \\ CS_{n-1}(2A) & 1/2C_n(2A) \end{bmatrix}. \quad (14)$$

$C_n(2A)$ and $S_{n-1}(2A)$ are Chebyshev polynomials of the first and second kind normalized to $2A$ rather than A as in (13). Term-by-term identification of (13) and (14) serves to define the Chebyshev polynomials C_n and S_{n-1} in terms of Chebyshev polynomials T_n and U_{n-1}. The Chebyshev polynomials $C_n(x)$ and $S_n(x)$ are available [2] for $n=2(1)12$ (*i.e.*, from 2 to 12 in steps of one) for $x=0(0.001)2$ to 12 decimal places. In this reference $U_{n-1}(x)$ is employed where $U_n(x)$ is usually found [4], [5].

A useful quantity for reflection loss and the resulting standing-wave ratio of a periodic transmission line structure is the equivalent susceptance B_{eq} which would give the same VSWR or insertion loss in a unity impedance line. It is found by $jB_{eq}=B-C$ (15) for one stage or $jB_{eq}=(B-C)S_{n-1}(2A)$ (16) for n stages; the relation to voltage standing-wave ratio r is

$$B_{eq} = \frac{r-1}{\sqrt{r}} \quad (17)$$

and to power insertion loss is

$$L = 1 + \frac{B_{eq}^2}{4}. \quad (18)$$

These relations are valid in both the pass band $|A| \leq 1$ and the stop band $|A| \geq 1$.

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X-Band Diode Limiting*

Broad-band, matched, low power, instantaneous, passive, X-band diode limiting has been demonstrated. The limiter, which uses standard microwave components, is an outgrowth of point contact germanium diode microwave switch research. In fact, the hybrid-tee switch[1] makes a very good narrow-band limiter under the condition of zero bias voltage on the crystals (biasing terminals short circuited). The output power under these conditions is limited to 0.5 mw[2] for incident power up to 30 mw, deduced from Fig. 9 of Garver, *et al.*[1] However, the bandwidth of the hybrid-tee switch when used as a limiter is insufficient for many applications such as limiting the amplitude of a 0.2-μsec magnetron pulse or flattening a frequency-modulated klystron mode.

From our experience with the hybrid-tee switch, we concluded that any diode switch providing high isolation with diode conduction and low insertion loss with nonconduction will function passively as a limiter. Low RF power does not cause significant diode conduction, while high RF power results in conduction which changes the diode impedance, causing increasing attenuation. Thus a more broad-band switch is needed that provides high isolation with the diode in the conducting state.

Although the basic X-band diode switch using a point contact germanium diode such as the 1N263 is broad-band, it provides high isolation in the nonconduction state, and therefore does not fulfil the above criteria. In fact, with its biasing terminals short-circuited, this switch acts as an expander.

Using a technique developed by Sweet,[3] it is possible to make a broad-band switch having isolation with diode conduction and which is matched as well for all biases on the diode. Fig. 1 shows the block diagram for using Sweet's technique at X-band. If both arms containing diodes are identical reflectors, the properties of the 90°, 3-db coupler are such that the phases of the reflected waves add at the output arm and cancel at the input arm; thus all power reflected from the diodes comes out the output arm, and the input arm always appears matched. Using slide screw tuners next to the diodes, a switch has been made giving isolation greater than 20 db over a 400-Mc bandwidth with an insertion loss of 0.7 db or less. The isolation is greater than 30 db over a 150-Mc bandwidth. This is much better than the hybrid-tee switch isolation of greater than 30 db over a 20-Mc bandwidth.

When the short-slot hybrid junction switch with two diodes is used as a limiter, however, the output does not remain constant with increasing incident power but increases slightly. The action could be better described as compression. Better limiting is obtained by using power sensitive tuners behind the diodes instead of fixed tuners. A second 1N263 space $\frac{3}{4}\lambda_g$ from the first has been found to give the flattest limiting. As shown in Fig. 2, the output at the center frequency is limited to 0.43 mw$\pm$0.1 db for all incident power from 2 mw to 200 mw.[4] For pulsed power up to 10 watts peak, the output increases to 2 mw peak. Pulse energy greater than 1 watt μsec permanently damages the diodes so that the low power insertion loss of the limiter is increased above its

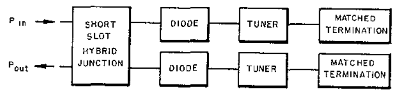

Fig. 1—Block diagram for making matched diode switch.

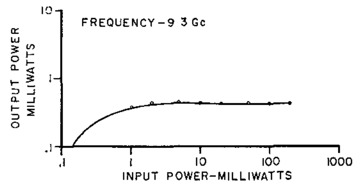

Fig. 2—Limiting using 4 1N263's and a short-slot hybrid junction.

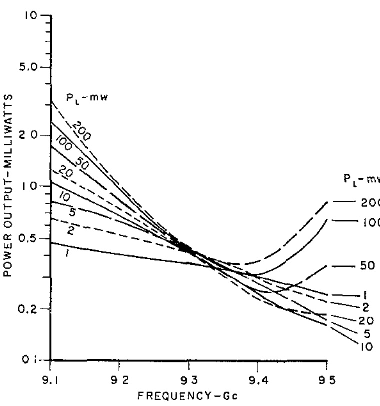

Fig. 3—Frequency dependence of limiting with diodes at $\frac{3}{4}\lambda_g$ spacing.

typical value of 0.7 db.

The frequency dependence of the limiting for $\frac{3}{4}\lambda_g$ spacing is shown in Fig. 3. For input power ranging from 2 mw to 200 mw, the output power is 0.45 mw$\pm$1 db over a 60-Mc bandwidth, and $\pm$2 db over a 120-Mc bandwidth. This limiter has been successfully used for suppressing the unwanted AM from an FM klystron.[5] Unless fringing effects forbit it, flat limiting should occur at one quarter wavelength between diodes. With the smaller spacing it is anticipated that only the abcissa of Fig. 3 would be changed, so that the data for 9.1 Gc would occur at 8.7 Gc and the data for 9.5 Gc would occur at 9.9 Gc, *i.e.*, the bandwidth should be tripled. To place the diodes one quarter-wavelength apart it will be necessary to redesign the diode mount so that it is smaller. Since the limiter does not require external biasing terminals or RF chokes, the dc shorts can be built into the diode mounts.

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* Received by the PGMTT, January 5, 1961.

[1] R. V. Garver, E. G. Spencer, and M. A. Harper, "Microwave semiconductor switching techniques," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 378–383; October, 1958. (See especially page 381.)

[2] Open-circuit bias terminals cause limiting at a higher output power level.

[3] L. Sweet, "Instantaneous Automatic Gain Control (IAGC) Techniques for Crystal Video Receivers," PRD Final Rept. 4.13, Contract No. AF 30(602)-1434/ Proj. No. 4505, Task No. 45215, ASTIA AD 148728; December 1, 1957.

[4] To obtain such flat output, it was necessary to use selected diodes. Diodes picked at random will give about 1 db variation in output. The characteristics were also slightly affected by diode rotation and seating.

[5] J. Samuel, "Diamond Ordnance Fuze Laboratories Workbook No. 1955." Unpublished.